# Lecture 2 10 Sep 07

## Last time

| me \ my pair | $\alpha$ | $\beta$ |
|---|---|---|
| $\alpha$ | $B^-, B^-$ | $A, C$ |
| $\beta$ | $C, A$ | $B^+, B^+$ |

outcome matrix

outcomes ≠ payoffs
payoffs matter

possible payoffs

| me \ my pair | $\alpha$ | $\beta$ |
|---|---|---|
| $\alpha$ | 0, 0 | 3, -1 |
| $\beta$ | -1, 3 | 1, 1 |

payoffs if only care about own grade
$\alpha$ strictly dominates $\beta$

**Lesson** Do not play a strictly dominated strategy

**Lesson** Put yourself in others' shoes to figure out what they will do

## Prisoners Dilemma

examples

| | |
|---|---|
| - joint project | incentive to shirk |
| - price competition <<collusion>> | incentive to undercut price |
| - common resource | incentive to "overfish" or pollute |

## Remedies

not joint communication

- contracts, treaties, regulations — change payoffs
- repeated play, education — change payoffs

<<Magic>>

## <<Today>>

**Formal Stuff** : Ingredients of a game

| | notation | # in game |
|---|---|---|
| • players | $i, j$ | you all |
| • strategies | $s_i$ a particular strategy of player $i$ | 13 |
| | $S_i$ the set of possible strategies of player $i$ | $\{1, 2, 3, \cdots, 100\}$ |
| | $s$ a particular play of the game | the spreadsheet |

a strategy profile
<<or vector, or list>>

• payoffs $u_i(s_1, \cdots, s_i, \cdots, s_N)$ $u_i(s) = \begin{cases} \$5 - \text{error} & \text{if wi} \\ 0 & \text{otherwis} \end{cases}$
$u_i(s)$

**Assume known**

one more notation

$s_{-i}$ a choice for all except person $i$

$u_i(s_i, s_{-i})$ <<useful way to think of this:

| 1 \ 2 | L | C | R |
|---|---|---|---|
| T | 5, -1 | 11, 3 | 0, 0 |
| B | 6, 4 | 0, 2 | 2, 0 |

players 1, 2
strategy sets $S_1 = \{T, B\}$ $S_2 = \{L, C, R\}$
payoffs eg $u_1(T, C) = 11$
$u_2(T, C) = 3$

**Defn** player $i$'s strategy $s_i'$ is strictly dominated by player $i$'s strategy $s_i$ if

$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ for all $s_{-i}$

payoffs <<"Hannibal">>

| defender \ attacker | e | h |
|---|---|---|
| E | 1,1 | 1,1 |
| H | 0,2 | 2,0 |

<< e,E = easy , h,H = hard >>
<< payoffs are how many battalions he'll arrive with >>
<< no dominant strategy >>
<< why'd they all choose E ? >>

**Defn** player $i$'s strategy $s_i'$ is **weakly dominated** by her strategy $s_i$ if

$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_{-i}$

$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ for some $s_{-i}$

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<< Game from last time >>

| | | |
|---|---|---|
| >67 | weakly dominated <<by 67>> | rationality |
| $67 \geq s_i > 45$ | not weakly dominated in original game, but weakly dominated once we delete 68-100 "in shoes" | rationality, + knowledge that others are rational |
| $45 \geq s_i > 30$ | "in shoes, in shoes" | R, KR, KKR |
| $30 \geq s_i > 20$ | "in shoes, in shoes, in shoes" | R, KR, KKR, KKKR |
| ⋮ | | |
| 1 | | Common Knowledge |

Rationality - takes out >67

<< Average $13\frac{1}{3}$ >>
<< $\frac{2}{3}$ Average 9 >>

<< It is mutual knowledge that someone wears a pink hat,
but not common knowledge

mutual $\not\Rightarrow$ common >>